33.27 **$1000cfm$ of fresh air is supplied to a room by a $16in$ diameter main duct which splits into two smaller ducts, A & B. Duct A is $13in$ and has an equivalent length of $50ft$. Duct B is $11in$ and has an equivalent length of $30ft$. Assuming the friction factor is consistent throughout the system, what percentage of the volume flow will travel through Duct A?**

A. 37%

B. 46%

C. 54%

D. 63%

Although the ducts do not recombine after splitting, because they serve the same space, this scenario is governed by parallel flow. Therefore, the friction loss for the two ducts will be equivalent. Write the Darcy-Weisbach Equation for both ducts and set the losses equal.

$$h_{f,A} = h_{f,B}$$

$$\frac{fL_A v_A^2}{2D_A g} = \frac{fL_B v_B^2}{2D_B g}$$

Cancel the friction factor, f, and the constants, 2, and g from both sides. Solve for the ratio of the velocities. Substitute known duct lengths and diameters and evaluate the ratio.

$$\frac{L_A v_A^2}{D_A} = \frac{L_B v_B^2}{D_B}$$

$$\frac{v_B}{v_A} = \sqrt{\frac{L_A D_B}{L_B D_A}} = \sqrt{\frac{(50ft)\,(11in)}{(30ft)\,(13in)}} = 1.19$$

$$v_B = 1.19 v_A$$

Write an expression for the total volume flow rate, which is given, as the sum of the volume flow rates through both branches expressed as the product of their respective velocities and areas. Consider working out the areas separately in advance.

$$A_A = \frac{\pi}{4}\left(\frac{13in}{12\frac{in}{ft}}\right)^2 = 0.922ft^2$$

$$A_B = \frac{\pi}{4}\left(\frac{11in}{12\frac{in}{ft}}\right)^2 = 0.660ft^2$$

$$Q_t = Q_A + Q_B = v_A A_A + v_B A_B$$

$$1000\frac{ft^3}{min} = v_A\left(0.922ft^2\right) + 1.19v_A\left(0.660ft^2\right)$$

$$1000\frac{ft^3}{min} = \left(1.71ft^2\right)v_A$$

$$v_A = 586\frac{ft}{min}$$

Find the volume flow rate through branch A. Calculate the percentage of the total volume through both branches.

$$Q_A = v_A A_A = \left(586\frac{ft}{min}\right)\left(0.922ft^2\right) = 540\frac{ft^3}{min}$$

$$\frac{Q_A}{Q_t} = \frac{540\frac{ft^3}{min}}{1000\frac{ft^3}{min}} = 54\%$$

Answer C